Séminaire Fables Géométriques

Tropical formulae for the number π

20th September

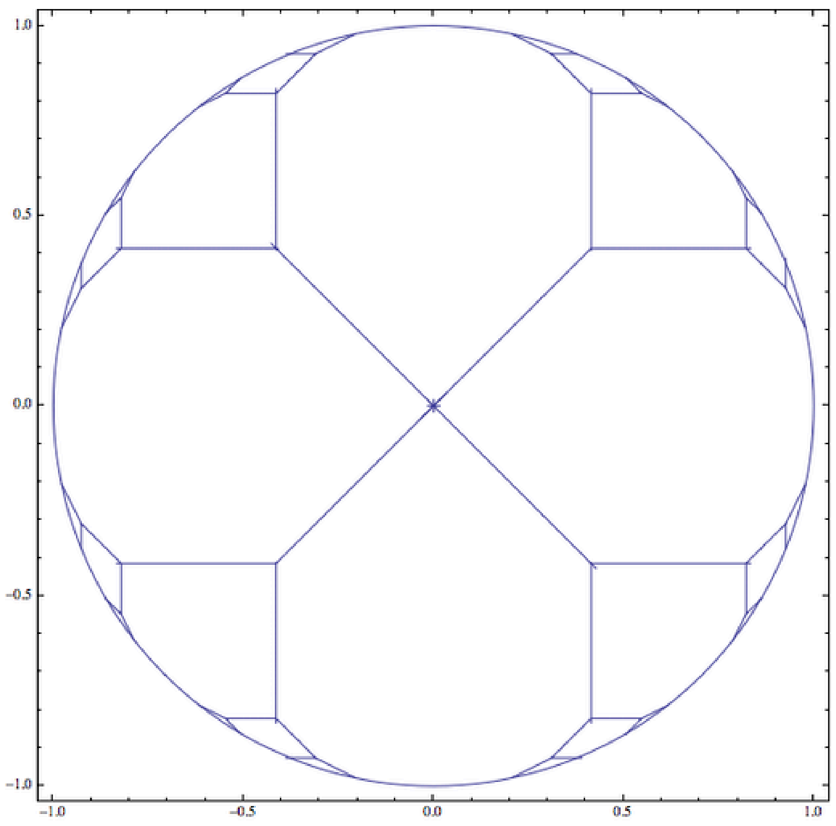
Nikita Kalinin

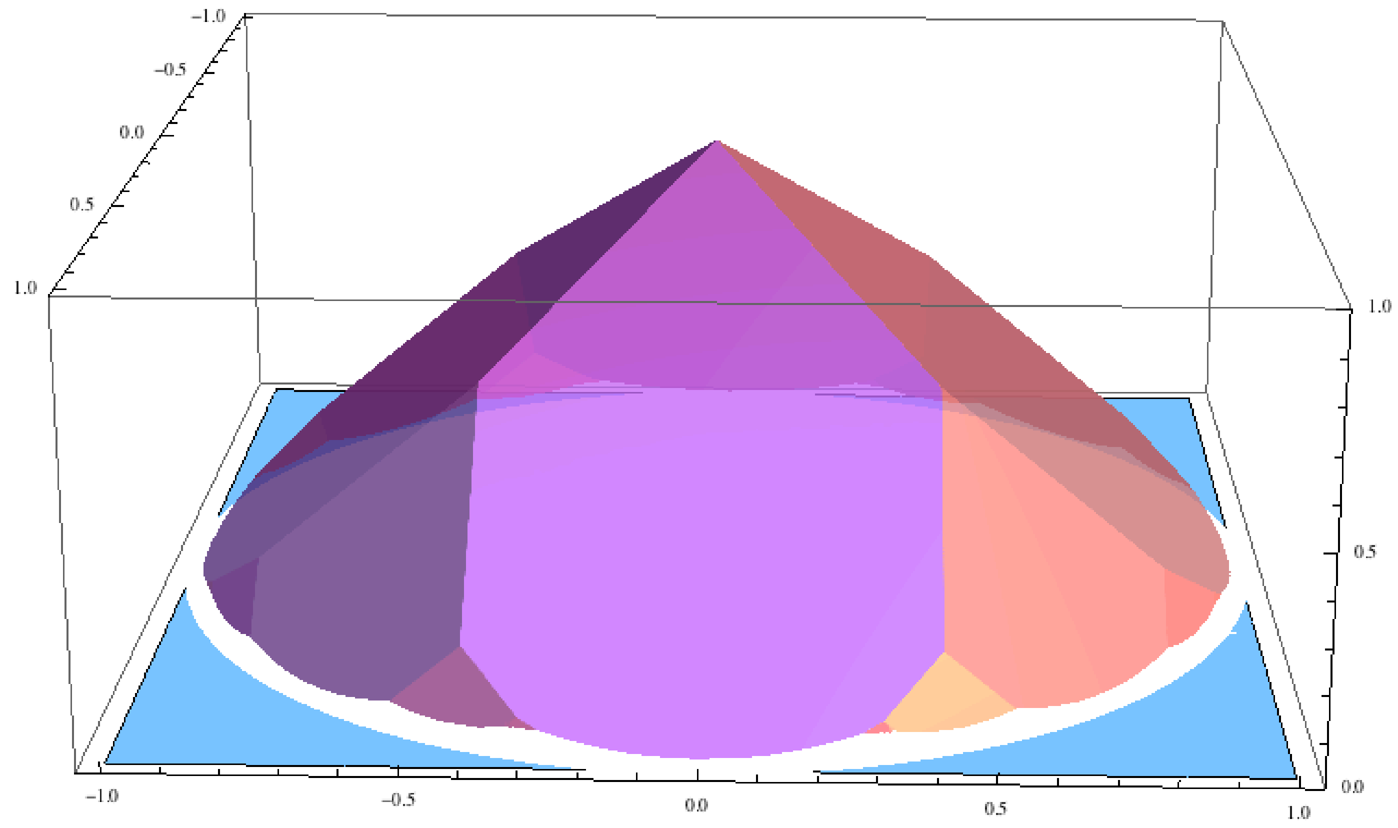
(Guangdong Technion Israel Intitute of Technology, China)

14:00

UNIGE, Section of Mathematics
Room 1-07

We will see how the same infinite series appear in 1) counting areas in between of Ford circles, 2) studying tropical caustics of convex domains 3) expansion a function in a certain Shauder basis associated with Farey fractions. Joint work with M. Shkolnikov.





$$f(a,b,c,d) = \sqrt{a^2+b^2} + \sqrt{c^2+d^2} - \sqrt{(a+c)^2+(b+d)^2},$$

$$\sum f(a,b,c,d) = 2, \quad \sum f(a,b,c,d)^2 = 2 - \pi/2.$$

$$4\sum\left(a\cdot\arctan(\frac{a}{b}) + c\cdot\arctan(\frac{c}{d}) - (a+c)\cdot\arctan(\frac{a+c}{b+d})\right)^2 = \pi.$$

$$\sum\left(\ln(\frac{\sqrt{a^2+b^2}^{b}\sqrt{c^2+d^2}^{d}}{\sqrt{(a+c)^2+(b+d)^2}^{b+d}}) + \ln\frac{(b+d)^{b+d}}{b^b d^d}\right)^2 = \pi.$$