Mathematics Colloquium

19 February
16h15

Room: 1-15
Section of mathematics
rue du Conseil-Général 7-9

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Building surfaces from equilateral triangles

Abstract. The construction of conformal maps of the earth, i.e. those that faithfully depict angles and orientations, has a long history. The most famous examples are given by stereographic projection (used since antiquity) and Mercator projection (constructed in 1569 and still used for internet maps today). One may also map a tetrahedron conformally to the sphere, taking care to specify what this means at the vertices. In 1965, cartographer L. P. Lee proposed using this classical fact as the basis for a map projection with less dramatic area distortion than the classical projections.

While this has not caught on, it suggests the following mathematical question: Which orientable surfaces can be conformally represented on a collection of equilateral triangles? Equivalently, which such surfaces can be built (up to a conformal change of coordinate) by glueing together a finite or infinite collection of copies of a closed equilateral triangle? Such surfaces are called *equilaterally triangulable*.

The answer in the compact case is given by a famous classical theorem of Belyi, which states that a compact Riemann surface is equilaterally triangulable if and only if it is defined over a number field. These *Belyi surfaces* - and their associated “dessins d’enfants” - have found applications across many fields of mathematics, including mathematical physics.

In joint work with Chris Bishop, we give a complete answer of the same question for the case of infinitely many triangles (i.e., for non-compact Riemann surfaces). The talk should be accessible to a general mathematical audience, including postgraduate students.

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